O K L A H O M A   S T A T E   U N I V E R S I T Y

SCHOOL OF ELECTRICAL AND COMPUTER ENGINEERING
SCHOOL OF MECHANICAL AND AEROSPACE ENGINEERING

ECEN/MAE 5713 Linear Systems
Spring 2012
Final Exam

Choose any four out of five problems.
Please specify which four listed below to be graded:
_____; _____; _____; _____;

Name: ______________________________

E-Mail Address:_______________________________________

Problem 1:

Find a minimal *observable* canonical form realization (i.e., its simulation diagram and state space representation) for the following MISO system described by

$$H(s) = \begin{bmatrix} \dfrac{2s+3}{s^3+4s^2+5s+2} & \dfrac{s^2+2s+2}{s^4+3s^3+3s^2+s} \end{bmatrix}.$$

Problem 2:
Let

$$S = \left\{ x \in \Re^3 | x = \alpha \begin{bmatrix} 0.6 \\ 1.2 \\ 0.0 \end{bmatrix} + \beta \begin{bmatrix} 0.5 \\ 1.0 \\ 0.0 \end{bmatrix}, \alpha, \beta \in \Re \right\},$$

find the orthogonal complement space of S, $S^{\perp} (\subset \Re^3)$, and determine an orthonormal basis and dimension for $S^{\perp}$. For $x = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}^T (\in \Re^3)$, find its direct sum representation (i.e., x_1 and x_2) of $x = x_1 \oplus x_2$, such that $x_1 \in S, x_2 \in S^{\perp}$.

<u>Problem 3</u>:

Given is the system of first-order ordinary differential equation $\dot{x} = t^2 Ax$, where $A \in \Re^{n \times n}$ and $t \in \Re$. Determine the state transition matrix $\Phi(t, t_0)$ and its solution, $x(t)$.

Problem 4:

Prove that $B(t) = \Phi(t,t_0) B_0 \Phi^*(t,t_0)$ is the solution of

$$\frac{d}{dt} B(t) = A(t)B(t) + B(t)A^*(t), \quad \text{with initial condition } B(t_0) = B_0,$$

where $\Phi(t,t_0)$ is the state-transition matrix of $\dot{x}(t) = A(t)x(t)$ and $\Phi^*(t,t_0)$ is the complex conjugate of $\Phi(t,t_0)$.

Problem 5:
Consider

$$\dot{x} = A(t)x + B(t)u$$
$$y = C(t)x$$

and its adjoint system

$$\dot{z} = -A^T(t)z + C^T(t)v$$
$$w = B^T(t)z$$
.

Let $G(t,\tau) = G_a^T(\tau,t)$ be their impulse response matrices. Show that

$$G(t,\tau) = G_a^T(\tau,t).$$